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Characteristics of Solar Irradiation

Solar Spectrum

The energy in solar irradiation comes in the form of electromagnetic waves of a wide spectrum. Longer wavelengths have less energy (for instance infrared) than shorter ones such as visible light or UV.

The spectrum can be depicted in a graph, the spectral distribution, which shows the relative weights of individual wavelengths plotted over all wavelengths, measured in W / m (wavelength).

The diagram displays the spectrum of a sun ray just outside the entry into the earth's atmosphere. The peak of the spectrum is within the visible spectrum, but there are still significant amounts of shorter and longer wavelengths present.

Intensity and Energy

For the purpose of solar power, the most significant measures are the intensity and energy delivered – one measure at a point in time, the other over a period of time.

At a point in time

Irradiance [W/m²]: The intensity of solar radiation hitting a surface, which is the sum of the contributions of all wavelengths within the spectrum, expressed in units of Watts per m² of a surface.

Power [W]: Momentary total irradiance incident on a particular area.

Over a period of Time

Energy per unit area [kWh/m²]: Energy per unit area is a measure of irradiance incident on a surface over a period of time. It is often expressed

Surface Orientation

As sunlight is smoothly distributed over whole areas, a mere figure for intensity is never sufficient without knowledge of the orientation of the surface in question. Typically, the orientation of a surface is described by the zenith angle, the angle between the sunbeam and the normal of the area. If the surface area is not perpendicular to the sunbeam (i.e. zenith angle is not zero), a larger area is required to catch the same flow as the cross section of the sunbeam.

If \( I_0 \) denotes the intensity on a surface with the sun in its zenith, the intensity, \( I \), on an area where the sun is observed under the zenith angle \( \theta \) (see figure) the intensity is reduced to

\[
I(\theta) = I_0 \cos(\theta)
\]

Values for \( \theta \) range from 0° to 90°. Turning the face of the area away from the sun means less energy is flowing through that area.

<table>
<thead>
<tr>
<th>Orientation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal Surface</td>
<td>Surface that lies flat on the ground of the earth</td>
</tr>
<tr>
<td>South facing surface</td>
<td>The projection of the normal of the surface onto the ground points to South.</td>
</tr>
<tr>
<td>Perpendicular surface</td>
<td>Surface that is perpendicular to the sunbeam with sun in zenith at ( \theta = 0° ).</td>
</tr>
</tbody>
</table>
Extraterrestrial Irradiation

The Solar Constant

The intensity of solar irradiation directly outside the earth’s atmosphere on a horizontal surface is almost constant at around 1,350 \( \text{W/m}^2 \), the so-called “Solar Constant”.

This entry point into the atmosphere is called “Air Mass – 0” or “AM-0”, where “0” points out that there is no air mass.

There is a variation of solar intensity of about 1%, but this is a slow cycle. It is so small that it is negligible for the purpose of solar power.
Modelling Solar Irradiation

With planetary movements, processes in the atmosphere and other effects, solar radiation on earth is an intermittent source of energy.

On the earth’s surface the peak solar intensity hovers around 1 kW/m² on a horizontal surface at sea level with the sun in its apex on a clear day. In general, the value will depend on the position of the sun, the clearness of the sky and the geometry of the surface.

Due to the complex nature of some of the processes, no theoretical calculations for irradiance is entirely accurate. Nevertheless, these models are helpful in understanding the main drivers as well as:

- Assisting in sizing of systems
- Aiding in choice of technology, as some technologies are more appropriate in certain locations than others.
- Forecasting energy generation
- Use before or in lieu of detailed site survey.
- Optimizing the design of devices and operations - especially predicting short-term variations of irradiance in the 1 - 10 minutes forecasting timeframe.

Mean intensity on horizontal surface on earth without atmosphere

Assuming the atmosphere has no impact on the incoming light, we can easily calculate a mean intensity on earth by dividing the total irradiance on the cross section of the earth by its surface area.

Calculation

Given the diameter of the earth, D=12,800km, the cross section is \( C = \frac{1}{4} \pi D^2 \). The intensity on the cross section is the solar constant of 1,350W/m². Hence, the mean intensity on the surface of the earth is \( I = \frac{I_0 C}{\pi D^2} \). This value already takes into account that at any one time, the sun only shines on half of the surface of the earth.

Result

<table>
<thead>
<tr>
<th>Intensity [W / m²]</th>
<th>Calculated Mean</th>
<th>Empirical Value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>342</td>
<td>187</td>
</tr>
</tbody>
</table>

Over a 24h period, this results in an average daily energy of 8.22 kWh/m².

Although this is a very crude approximation, it provides a rough estimate for the absolute limit of solar energy on earth. However, this simplistic model fails to describe the impact of:

- Variations over time due to planetary movements and activities in atmosphere.
- Attenuation of light due to absorption in atmosphere.
- Difference between diffuse and direct light.
- Effect of latitude, tilting of surfaces and reflection off the ground.

In this document, we are presenting basic models for the effects of the solar orbit, tilting surfaces, earth's rotation as well as attenuation and absorption in the atmosphere.
The Solar Orbit

Due to the eccentricity of the earth's orbit around the sun, the intensity of the solar radiation just outside the earth's atmosphere is not constant. In fact, the solar constant, $I_0$, is the annual average intensity. Between January 2nd when the earth is closest to the sun and 3rd July when it is furthest away, the intensity varies ±3% from the average.

In mathematical terms it is:

$$I(n) = I_0 \left[ 1 + 0.034 \cos \left( \frac{2n}{365} \right) \right]$$

with $n$ denoting the day of the year, $n = 1...365$. As a consequence, the Southern hemisphere enjoys more irradiation during its summer than the Northern hemisphere.
**Geometrical Aspect: The Sun's Observed Position**

The angle under which the sun is observed from a point on the earth's surface is affected by the earth's daily rotation, the annual rotation of the tilted earth and the latitude of the surface in question.

**Daily Rotation**

The solar hour angle $\Omega$ expresses the daily rotation of the earth. As the earth rotates 360° within 24 hours, every hour adds another 15° to the solar hour angle. When the sun is in its highest point in the sky, the solar hour angle is zero. Angles before noon count negative, after noon positive.

As the earth rotates, the angle between the sun and due south, the solar azimuth angle, $\alpha$, varies from -90° at sunrise (east) to +90° at sunset (west).

![Diagram: Solar Azimuth Angle $\alpha$](image)

**Declination**

The declination angle, $\delta$, is the angular position of the sun at solar noon with respect to the plane of the equator. It varies from -23.45° at winter solstice to +23.45° at summer solstice according to:

$$
\delta = 23.45 \frac{\pi}{180} \sin \left( 2 \pi \frac{284 + n}{365} \right)
$$

As before, $n$ denotes the day in the year with $n = 1..365$.

**Latitude**

For locations other than the equator, the latitude, $\phi$, changes the zenith angle further.

**Zenith Angle**

The orientation of an area with respect to the sun is best described by the zenith angle, $\theta$, which is related to latitude, day and time via:

$$
\cos(\theta) = \sin(\phi) \sin(\delta) + \cos(\phi) \cos(\delta) \cos(\Omega)
$$

To get the intensity, the peak intensity should be multiplied with the cosine of the zenith angle.

The above equation is only true while the sun can be observed, which allows us to determine the hours of daylight on a horizontal surface as

$$
\text{Hours} = \frac{24}{\pi} \arccos(-\tan(\phi) \tan(\delta))
$$

Obviously, averaged over the year, every location on earth has the same amount of hours of daylight. However, the annual distribution varies by latitude. While cities with higher latitude enjoy more hours of daylight in summer, the peak intensity remains lower due to the larger zenith angle.
Graphing Intensity and Energy

Using the above formulae, we have put together a few graphs to demonstrate the impact of latitude, time of day and day of year on the intensity and energy observed on horizontal surfaces. All graphs have been normalised to 1kW / m².

Daily hours of sunlight over the year

Intensity on horizontal surface on 22 June during 24 hours
Intensity on horizontal surface at 5AM over the year

Intensity on horizontal surface at 12 midday over the year

Daily energy on horizontal surface over the year
Effects in the Atmosphere

In various layers of the atmosphere, the sun light gets reflected back outside, absorbed and scattered. As a result, the radiation that hits a horizontal surface on the earth is

- attenuated,
- split into diffuse and direct light, and
- has a different spectrum to the one outside the atmosphere.

**Attenuation**

To express the amount of intensity that is lost through absorption, the *clarity index* is defined as the ratio between the observed (global) hourly irradiance on earth, $H_g$, and the hourly radiation $H_0$ just outside the atmosphere:

$$K = \frac{H_g}{H_0}$$

The actual values for $K$ have to be measured. Typical values are:

<table>
<thead>
<tr>
<th>Condition</th>
<th>$K$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clear sky at sea level</td>
<td>0.6 – 0.8</td>
</tr>
<tr>
<td>Cloudy</td>
<td>0.1 – 0.3</td>
</tr>
</tbody>
</table>

Around 18% of the extraterrestrial radiation is absorbed or reflected back. Higher latitudes experience lower values, as the path through the atmosphere under a larger zenith angle is much longer.

The clarity index is usually either daily or hourly to average out short-term fluctuations. It is assumed that clouds are uniformly distributed over the sky. Drifting clouds are not considered in this technique.

**Diffusion**

Direct beam radiation is one which strikes the surface from one angle only, directly from the sun. Conversely, diffuse light, as a result of absorption and scattering, approaches the horizontal surface from almost any angle. It can therefore not be focused or concentrated.

The global hourly irradiance on a surface can be expressed as the sum of direct (beam) and diffuse radiation as:

$$H_g = H_{\text{Beam}} + H_{\text{Diffuse}}$$

Similar to the clearness index, the *diffusion index* is defined as

$$K_{D} = \frac{H_{\text{Diffuse}}}{H_g}$$

The *beam fraction* is $1 - K_D$.

We have plotted the typical relationship between the beam fraction and the clearness index for latitudes around 50°N. As expected, clear skies cause less diffusion. However, where there are clouds, the ratio of diffuse light can be in excess of 75%. Any devices that concentrate light onto a single point rely on a high proportion of direct beam and are therefore not suitable in locations with high diffusion index.
Radiation on Tilted Surface

In addition to direct beam and diffuse light, a tilted surface will also be struck by rays reflected off the ground. Accordingly, the radiation on a tilted surface has three components:

\[
H_{\text{Beam}}^T = H_{\text{beam}}^T + H_{\text{Diffuse}}^T + H_{\text{Reflected}}^T
\]

**Beam Radiation**

If \( RB \) denotes the ratio of the average daily beam radiation on a tilted surface to that on a horizontal surface, then the direct beam part can be written as

\[
H_{\text{Beam}}^T = H_{\text{Beam}} \cdot RB
\]

\( RB \) is a pure geometric parameter, dependent on the horizontal tilt, surface azimuth, declination angle and latitude.

**Diffuse Radiation**

Assuming an isotropic distribution of the diffuse radiation over the hemisphere, the diffuse part is only dependent on the horizontal tilt angle \( \beta \) and the diffuse radiation of the horizontal surface:

\[
H_{\text{Diffuse}}^T = \frac{H_{\text{Diffuse}}}{2}
\]

This takes into account that the tilted slope sees only a portion of the hemisphere.

**Reflected Light**

The energy of the reflected light is dependent on the ground’s ability to reflect, a property which is expressed by the albedo factor \( \rho \). The albedo ranges from 0.1 (asphalt paved road) to 0.9 (snow). Given the albedo, the reflected term can be calculated from:

\[
H_{\text{Reflected}}^T = \rho (H_{\text{Beam}} + H_{\text{Diffuse}}) \cdot \frac{1 - \cos(\beta)}{2}
\]

<table>
<thead>
<tr>
<th>Material</th>
<th>Albedo</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lawn</td>
<td>0.205</td>
</tr>
<tr>
<td>Untilted Field</td>
<td>0.26</td>
</tr>
<tr>
<td>Naked Ground</td>
<td>0.17</td>
</tr>
<tr>
<td>Weather-beaten concrete</td>
<td>0.3</td>
</tr>
<tr>
<td>Asphalt</td>
<td>0.15</td>
</tr>
<tr>
<td>Fresh snow</td>
<td>0.85</td>
</tr>
<tr>
<td>Old snow</td>
<td>0.58</td>
</tr>
</tbody>
</table>

The reflected light itself has parts of diffuse and parts of direct light.
Radiation on tilted surface in relation to horizontal surface

It can be shown with the help of the above formulas that tilting up a surface can increase the irradiance incident. The actual amount depends on numerous factors such as latitude, day in the year, albedo and clearness index as well as both the tilting angle and the surface azimuth.

We have plotted the irradiation components relative to the global irradiation on a horizontal surface facing due south on 20th March at 50°N with an albedo of 30% on a reasonably clear day (clearness index 0.5). Under these circumstances, the optimal tilt angle would be around 40°.

Intuitively, the tilting effect is more pronounced for higher latitudes.

Optimum Surface Orientation

Tracking
In order to maximise the direct-beam insolation on a surface, it is required to rotate the surface around two axes, namely the tilt and the azimuth angle, which requires two motors. Typically, the marginal energy gains from tracing the azimuth angle are low. Hence, the second best option is to keep the slope flexible, but facing due south.

Fixed Tilt
In case there is no possibility to move the surface at all, the optimal tilt angle for the maximum amount of direct beam radiation is equal to the site's latitude. Tilting the surface up, however, causes the diffuse light portion to decrease. The optimal tilt angle for sites with humid climates is therefore 10 – 25% less than the latitude. In Germany, for instance, at 48°N, a tilt angle of 30° would be optimal, whereas in Spain, it could be up to 40°.

Seasonal Tilt
In regions where most of the irradiance occurs in summer, it may be beneficial to adjust the tilt angle for winter and summer. In Germany, 75% of solar irradiance is experienced from April to September. The optimal angle for the summer would be 27° and for winter 50°, rather than 30° if the modules couldn’t be tilted at all. In Spain, seasonal differences are less pronounced (summer accounts for 60%), making a seasonal tilt less critical.
Annual Energy Yield, Intermittency and Annual Variations

While no model can fully replace actual measurements to fully assess the potential of a site, solar irradiance maps can provide a first insight. The following graphs show various maps with average annual energy values on fixed, due-south facing surfaces that are optimally inclined.

In addition, system designers and project developers should take into account effects of intermittency and annual variations.

World Map - Annual global versus direct irradiance

The following maps highlight the differences between direct and global irradiance where "global" includes diffuse light. Areas with a high proportion of diffuse light include Northern Europe, South-East China and the tropical belt around the equator. (Source: Meteonorm)
Europe - Annual global irradiance

South America - Annual global irradiance
Intermittency

If we assume a peak intensity of 960 W/m², the ratio of the average intensity to the peak intensity in Europe is between 12% (Iceland) – 24% (Southern Spain) with the average around 18%. This is a measure of the intermittency of the solar energy source. It also means that solar systems have to be built to cope with peak intensity, but will on average only be able to convert 18% of that peak. This factor is closely related to the so-called capacity factor.

Annual Variation of solar insolation

The energy (or insolation) received on a surface throughout the year varies relatively little from year to year.

Intuitively, more energy comes from the time intervals with high irradiation. The contribution of highly intense light can vary significantly from one year to another. The standard deviation of the overall annual energy, however, is around +/-4%.

I.e. the annual energy delivered by the sun does not vary greatly year on year. The sun provides a stable "traffic".

As an example, we have plotted the distribution of irradiance levels for 10 different years in Kassel, Germany (source: BP).
The atmosphere does not just change the overall intensity, but the whole spectral distribution. For instance, most of the high-energy wavelengths that are present in the sun light are filtered out by the ozone layer. Generally, with longer paths through the atmosphere (at higher latitudes or around sunset), the larger the part of infrared light, the low energy spectrum. This filter effect can be expressed by a turbidity factor.

In order to be able to compare solar modules, standard test conditions have been designed. These conditions include spectrum, intensity and temperature. The standard spectra refer to generic locations. They are prefixed “AM”, which stands for “Air Mass” and followed by a number, which refers to the length of the path through the atmosphere in relation to the shortest length if the sun was in the apex. It is roughly

$$\text{AM} = \frac{1}{\cos(\theta)}$$

with the zenith angle $\theta$.

The Committee Internationale d’Eclaireage (CIE) and the American Society for Testing and Materials (ASTM) publish a number of spectra. Their origins are from actual measurements, which are subsequently declared standard. They are also designed such that the spectrum can be reproduced artificially.
<table>
<thead>
<tr>
<th>Name</th>
<th>Standard</th>
<th>Conditions</th>
<th>Intensity W/m²</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>AM-0</td>
<td>ASTME E490</td>
<td>Just outside the earth’s atmosphere</td>
<td>1,353</td>
<td></td>
</tr>
<tr>
<td>AM-1</td>
<td>CIE Publication 85</td>
<td>Sun overhead, sea level, horizontal surface</td>
<td>1,000</td>
<td></td>
</tr>
<tr>
<td>AM-1.5G</td>
<td>ASTM G173-03</td>
<td>Tilted surface at 37°, zenith 48°, facing due south, albedo 0.3, turbidity 0.29, 20°C ambient temperature</td>
<td>963.8</td>
<td>Global Radiation – these conditions reflect the average of the 48 contiguous states of the US.</td>
</tr>
<tr>
<td>AM-1.5D</td>
<td></td>
<td></td>
<td>768.3</td>
<td>Direct Radiation</td>
</tr>
<tr>
<td>AM-1.5G</td>
<td>CIE</td>
<td></td>
<td>1,000</td>
<td>Same as AM-1.5G ATM E-892, however it is normalized to 1kW/m2 by simple multiplication.</td>
</tr>
<tr>
<td>AM-2</td>
<td></td>
<td>Zenith 60°.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
## Insights

### What the effects of solar radiation mean to solar power

Taking into account the various effects of solar radiation (e.g. intermittence, absorption, scattering, tilting, eccentricity), solar power converters, installations and yield forecasts can be optimised.

<table>
<thead>
<tr>
<th>Effect</th>
<th>Impact on Intensity</th>
<th>Insight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variations in solar activity</td>
<td>+/- 1%</td>
<td>Negligible effect, unavoidable</td>
</tr>
<tr>
<td>Eccentricity in solar orbit</td>
<td>+/- 3% over the year</td>
<td>Summer on the Southern hemisphere is more pronounced than on Northern hemisphere.</td>
</tr>
<tr>
<td>Planetary movements: Rotation and declination</td>
<td>Significant but predictable impact – lower intensity per m² when sun is not in zenith.</td>
<td>Although all locations on earth enjoy the same number of daylight, differences depending on latitude are significant, while effects of longitude cancel each other out.</td>
</tr>
<tr>
<td></td>
<td>24h cycle of day and night Annual cycle of summer and winter</td>
<td>Large variations, but entirely predictable.</td>
</tr>
<tr>
<td>Absorption in atmosphere</td>
<td>Further attenuation through atmosphere.</td>
<td>This happens even on a clear day.</td>
</tr>
<tr>
<td>Scattering in atmosphere</td>
<td>Splits light into direct and diffuse light.</td>
<td>There is more diffusion further away from the equator and in humid climates.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Diffuse light can not be focused (concentrated), but can still be used in the photovoltaic effect.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Diffuse light has a larger proportion of long waves (infrared), which may not be sufficient to cause the photovoltaic effect in Silicon.</td>
</tr>
<tr>
<td>Tilting of surface</td>
<td>Also receive reflection off the ground</td>
<td>By tilting a surface towards the sun, the ratio of direct light that strikes the surface increases significantly, also helped by reflection off the ground. In the Northern hemisphere, the optimal tilt angle for a fixed surface is 10% less than its latitude.</td>
</tr>
<tr>
<td>Intermittence</td>
<td>Average intensity is between 10 – 30% of peak intensity</td>
<td>Solar power converters have to be sized such that they can handle peak intensity, though on average they can only use 10 – 30% of that peak.</td>
</tr>
</tbody>
</table>